A New Monetarist Model of Fiat and E-Money*

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Abstract

We study the competition between fiat and electronic money (such as an e-purse) as a payment instrument. Our intent is to explain the reasons for the e-purse failure in Europe compared to its success in Asia and the U.S., and the conditions necessary for its greater adoption. E-money is safer than cash yet it is not universally accepted since merchants have to acquire the reading terminal needed to transfer e-money. We show that although buyers receive the storage device (e-purse) for free, they may decide not to use e-money. Additionally, although sellers must incur an investment cost to accept e-money, cash may be abandoned. Our framework captures the strategic complementarities between consumers and retailers leading to multiple equilibria where both payment instruments may be used, and we study the conditions for e-money to replace cash.

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1 Introduction

The past two decades can be characterized as a period in which there have developed a large variety of payment instruments made available to consumers. The increased use of these various payment instruments has resulted in a decline in the use of the more traditional methods of payment, both cash and checks. One of these instruments, electronic money, such as e-purse or prepaid cards, appeared in various countries in the 1990's specifically to replace cash in small value transactions.[1]

The European Central Bank (ECB) defines e-money as "a claim on the issuer that is (*a*) stored on an electronic device, (*b*) issued on receipt of funds of an amount not less in value than the monetary value issued, and (*c*) accepted as a means of payment by undertakings other than the issuer" (ECB, 2000). The Bank of International Settlements defines electronic money as "a stored value or prepaid product in which a record of funds or value available to the consumer for multipurpose use is stored on an electronic device in the consumers possession" (BIS, 2004). Consequently, electronic money acts as a prepaid instrument that can be considered an electronic substitute for coins and banknotes. In fact, electronic money is a bank liability to the issuer which circulates like paper money without bank authorization after emission, and which can be redeemed for one unit of fiat money. However, e-money cannot be re-used by the recipient who must first transfer e-money into his bank account to submit payment.

In order to reduce the cost of cash management, many initiatives have emerged around the world to provide individuals and merchants with an alternative to cash. Partisans of a cashless society state that technological innovations in payment instruments constitute a solution to reduce cash management costs, estimated at around 250€ a year, per person, in Europe (McKinsey Report, 2008). The electronic purse is the closest substitute for cash in everyday small-value transactions, and due to its recent emergence, some economists have predicted the imminent demise of cash for current settlements (Dowd, 1998, Friedman, 1999, King, 1999). However, in developed countries, e-money does not constitute a credible substitute for cash despite the high number of storage devices available for free at Universities, Staff Canteens, and the development of mandatory uses for parking meters, as an example. This situation contrasts with the success of reloadable transport and payment cards in Asia, and the increasing trend of e-purse adoption by the unbanked in the

[1] Mondex was launched in 1995 in the U.K., Proton in 1996 in Belgium, GeldKarte in 1998 in Germany, Chipknip in 1998 in the Netherland, Moneo in 1999 in France. Octopus was launched in Hong Kong in 1997, the EZ-Card in Singapor in 1998, and the Suica Card in Japan in 2001. In the U.S., Visa launched its first contactless prepaid card in 2006, and in 2008 the U.S. Department of the Treasury's Financial Management Service chose Comerica Bank as its financial agent to issue prepaid cards for the unbanked in order to receive Social Security, or personal income.

US.

The latest international statistics on settlement systems indicates that electronic money constitutes only 2.2% of the total number of transactions in 2012 (and 3% as a percentage of the total value transactions), although the number of transactions executed with it increased by 21.4%. Between 2011 and 2012, the e-money transactions increased by 3.6% compared to the number of cash withdrawals that increased by 13% over the same period, which can explain the recent decreasing trend of e-money use in some developed countries such as Belgium, Germany, or Switzerland (BIS, 2013).[2] Therefore, despite the existence of alternatives to the use of fiat money, cash remains the most widely used means of payment for everyday transactions in the retail sector. Our intention is to better understand, and explain this situation. To do so, we study the conditions under which electronic money can replace cash for payments when money is essential, i.e. in a Lagos and Wright (2005) search model.

In our model, we differentiate cash from electronic money in the following way. Cash is universally accepted by sellers at no cost but can be stolen at any time, so there is always the risk of theft when carrying fiat money. Electronic money is only partially accepted in that, in order to be able to pay with it, retailers must invest in an electronic device at the point of sale to accept e-money payments. Such a device is costly for sellers, yet always makes the payment safe. Money collected is electronically recorded in the reading terminal before being sent by communication networks to sellers bank accounts, to be credited[3]. For buyers, banks can also play this safekeeping role by guaranteeing the refund of e-money lost or stolen if agents have chosen a safe electronic means of storage which is personalized and associated with a bank account.

Carrying cash or e-money involves the same opportunity cost in terms of foregone interest earnings. However, these two means of payments are imperfect substitutes, and this is precisely why we analyze agents decisions to adopt or not to adopt the new payment instrument. On one hand, an important feature is the lack of universal acceptability of electronic money. It requires that buyers form expectations about the probability of being able to use their e-purse if they enter the market with it. However, sellers must decide whether or not to accept electronic money in parallel

[2] According to the ECB (2012), 65% of transactions in the EU-27 are settled in cash, while only 1.06 % were paid in electronic money. In comparison, the relative share of other payment instruments were 2.5% for checks, 25% for credit cards, 8% for direct taxes and 10% for loans. Therefore, the relative share of electronic money is marginal in Europe.

[3] The value spent by the buyer is reduced from his card balance and is added to the balance of the seller's reading machine. The seller redeems the monetary values at the end of the day from a clearing operation (like with credit cards) by sending to the network the stored values which are credited to his bank account (Lacker, 1996).

-or instead of- cash given the cost associated with the investment in an electronic reading terminal. The buyers and sellers joint decisions create strategic complementarities in the choice of the payment instrument, like in Nosal and Rocheteau (2011), and Lotz and Zhang (2015)[4]. Consequently, replacing cash with electronic money requires that the latter becomes a quasi-universal, or largely accepted means of payment.

We provide a framework to explain the conditions under which electronic money can replace cash. The cost of investment is important to understand sellers e-money adoption, but not sufficient. Indeed, sellers expectations about the buyers portfolio composition are crucial to give them incentives to invest. Two ingredients are also important for buyers to decide to carry e-money or not: the risk of theft associated with holding cash, and the probability of being able to use e-money in future transactions. Our analysis shows that when the risk of theft increases, the price of goods increases as it incorporates a risk premium paid to the seller for the risk associated with a cash payment, which increases e-money adoption. Consequently, different payment patterns can emerge, where fiat money coexists with electronic money, or where one of the two monies dominates the other.

The analysis of the buyers choice of a means of payment is not a recent issue, and has been developed in different frameworks based on transaction costs (e.g. Whitesell, 1992; Folkertsma and Hebbink, 1998; Shy and Tarkka, 2002). More recently, the competition between different payment instruments has been studied in search based environments which explicitly model trade frictions. Closely related to our paper is He et al. (2008), and Sanches and Williamson (2010), who model the circulation of cash and bank liabilities or credit, respectively, by considering buyers risk of theft of fiat money. Li (2011) studies the role of a fixed record keeping cost to distinguish checking deposits from currency, and considers that cash holdings are costly because of the forgone interest earnings. All these studies use the generalized Nash bargaining solution for the pricing mechanism. By contrast, we consider that each agent (buyer or seller) who holds cash may be stolen, and we use the proportional bargaining solution in order to give sellers an incentive to invest in the costly record keeping technology. Nosal and Rocheteau (2011), Lester et al. (2012), and Lotz and Zhang (2015), also assume an investment cost in a technology allowing sellers to accept credit, or assets, instead of cash. Contrary to these last studies, our two currencies have the same opportunity cost, and we introduce some risk associated with holding cash, instead of e-money, in the decentralized

[4]This characteristic is common to all two-sided markets (Rochet and Tirole, 2003).

market. This allows us to explain the tradeoff buyers and sellers face, which provides strategic complementarities between agents and implies multiple steady-state equilibria.

The remainder of the paper is outlined as follows. In section 2, we present the model. In section 3, we describe agents behavior in the centralized market. In section 4, we determine the terms of trade, depending on what buyers and sellers use and accept as means of payment. In section 5, we endogenously determine both the buyers portfolio composition (cash versus e-money) and the sellers investment decision in the e-money technology. In section 6, we study monetary equilibria. In section 7, we compare the results of our model with e-money experiences in some developed countries. Section 8 concludes.

2 Assumptions

Our model is based on the monetary search model proposed by Lagos and Wright (2005), and developed by Nosal and Rocheteau (2011). There is a $[0, 2]$ continuum of infinitely-lived agents in the economy, evenly divided between buyers (consumers) and sellers (producers), and time is discrete. Agents visit two markets during each period, which correspond to two sub-periods. In the first sub-period, agents enter a decentralized market, called DM. Anonymous buyers and sellers, specialized in consumption or production, are matched bilaterally and randomly. Sellers can produce an output, in quantity $q \in \mathbb{R}^+$, but do not consume, while buyers want to consume but cannot produce. In this model, barter is impossible, credit is ruled out, and goods are perishable. The fact that sellers do not want to consume and that buyers cannot produce generates a problem of double coincidence of wants that justifies the use of a means of payment. As a result, money is essential to trade.

In the DM, a buyer meets a seller, and *vice versa*, with probability σ. The value of this parameter summarizes the trading frictions in the market. After a meeting, if an agreement is reached, agents exchange an amount of a specific good, called search good, for money. Here, agents can either hold fiat money, or a new payment instrument, electronic money. Thus, in the DM, q units of goods are traded either against fiat money (f), electronic money (e), or both depending on the composition of the buyers portfolio, i.e. $q = q(f, e)$. However, at the end of DM, we assume that agents units of fiat money are exposed to risk of theft with probability $\alpha \in (0, 1)$, contrary to agents units of e-money which are fully secured.

At the end of the first sub-period, agents enter a frictionless centralized market that corresponds to the second sub-period, called CM since it is a competitive market. There, all individuals can consume or produce a general good $x \in \mathbb{R}^{+}$ by supplying labor (h). Usually, buyers produce the general good in order to increase their monetary balances spent in the previous market, and sellers consume the general good to reduce their money holdings. Additionally, buyers choose the composition of their portfolio for the following period, which may contain one or both types of money.

The utility functions of buyers (U^{b}) and sellers (U^{s}) for the entire period, assumed separable between the two sub-periods, are described as follows:

$$\begin{aligned} U^{b}(q, x, h) &= u(q) + x - h \\ U^{s}(q, x, h) &= -c(q) + x - h \end{aligned}$$

In the CM, agents utility function is linear with respect to labor hours: $U(x,h) = v(x) - h$, where $v(x) = x$ with no loss in generality. The utility and cost functions are assumed to respect the following properties: $u'(q) > 0, u''(q) < 0$, $c'(q) > 0, c''(q) > 0, u(0) = c(0) = c'(0) = 0$, and $u'(0) = +\infty$. The optimal quantity produced and exchanged corresponds to the level of production and consumption that maximizes agents trade surplus $u(q) - c(q)$. This quantity q^{*} solves $u'(q^{*}) = c'(q^{*})$. Finally, all agents depreciate the future between each period, but not between sub-periods, at the discount rate $\beta = \frac{1}{1+r} \in (0,1)$.

There are two means of payment in the economy, fiat money and electronic money, that are perfectly divisible and storable. Electronic money is a new means of payment that is intended to replace cash. The payments with electronic money are made possible by the following technology: sellers must have a payment terminal to accept e-money, and buyers must store e-money in an e-purse (or prepaid card) to transfer it to sellers. Buyers get the storage device for free, while the e-payment terminal is costly for sellers. A fraction $\Lambda \in [0,1]$ of sellers may decide to adopt this technology, which will allow them to accept both fiat and electronic money, whereas the remaining sellers will only be able to accept cash in exchange for goods. The fraction Λ, which is an endogenous variable, will be determined at the equilibrium by the sellers investment decision, and the buyers portfolio choice.

At the beginning of the period, each buyer enters the DM with a portfolio of money balances $(m = m_f + m_e)$ consisting of m_f nominal units of fiat money and/or m_e nominal units of electronic money, with $m_f \geqslant 0$, and $m_e \geqslant 0$. Monetary units are valued at $\phi_t = \frac{1}{p_t}$, ϕ_t being the price of money in terms of the general good, at time t. Consequently, the real value of the nominal portfolio

m is: $z = \phi(m_f + m_e) = f + e$, where $f = \phi m_f$, and $e = \phi m_e$. The aggregate stock of money in the economy is M, and can increase or decrease at a constant gross rate $\gamma = \frac{M_{t+1}}{M_t}$. Money is injected or withdrawn from the economy by the government through lump-sum transfers T to the buyers in the CM. Finally, we study stationary equilibria such that the real value of the money supply is constant over time: $\phi_t M_t = \phi_{t+1} M_{t+1}$.

Traditionally, monetary search models formalize four types of frictions: (i) search frictions in the decentralized market, (ii) anonymity of agents, (iii) lack of record keeping technology of agents actions, and (iv) limited commitment. Our assumptions involve two supplementary elements: (v) a risk of theft of fiat money holdings, and (vi) sellers investment cost in the e-payment terminal. The timing of events is described in Figure 1.

3 The centralized market

We begin with the centralized market (CM). All agents can produce and consume. They produce the general good with h units of labor, consume x units of a general good, and readjust their money balances, i.e. choose the type and quantity of money to hold before entering the following decentralized market (DM).

First, we consider the buyers maximization problem. The value function of a buyer $W_t^b(z)$ who

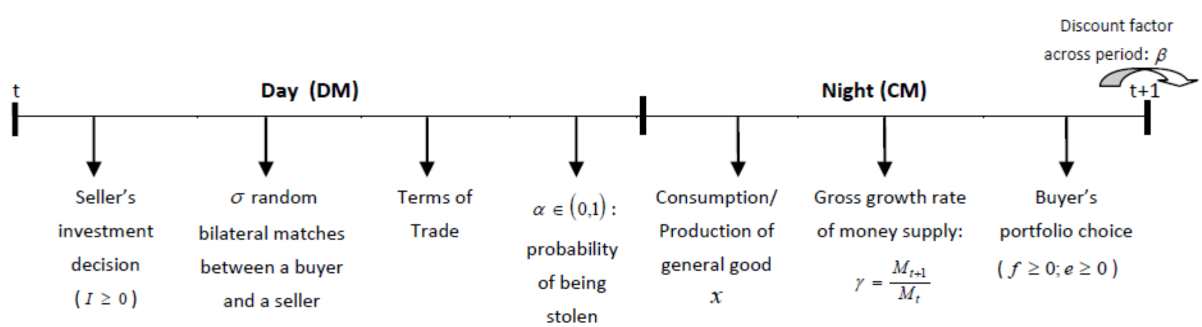

Figure 1: Timing

enters the CM with a portfolio of z real balances is:[5]

$$\begin{aligned} W_t^b(z) &= \max_{x,h,z'\geqslant 0}\left[x-h+\beta V_{t+1}^b(z')\right] \\ s.t.\ x+\phi_t\left(m_f'+m_e'\right) &= h+z+T \\ z' &= \phi_{t+1}\left(m_f'+m_e'\right)=f'+e' \end{aligned}$$

where V_{t+1}^b is the buyer value function in the following DM. The second equation is the buyer budget constraint. He finances his consumption of the general good (x) and his next period real balances (z') with labor (h), his current real balances (z), and lump sum transfers from the government (T). From the previous equations, the buyer value function at the beginning of the CM can be rewritten:

$$W_t^b(z)=z+T+\max_{f',e'\geqslant 0}\left[-\gamma\left(f'+e'\right)+\beta V_{t+1}^b\left(f',e'\right)\right] \tag{1}$$

According to this equation, the buyer lifetime expected utility when he enters the CM is the sum of his current real balances, the lump-sum transfers from the government, and his value function at the beginning of the following DM minus the cost of holding money (γ). As in Lagos-Wright (2005), the value function is linear with real money balances. Therefore, the choice of the buyer portfolio (f', e') is determined independently of his current real balances (z). Moreover, the linearity of the utility function eliminates wealth effects between periods.

Identically, the value function of a seller who holds z real balances at the beginning of the centralized market is given by $W_t^s(z) = z + \beta V_{t+1}^s$, where V_{t+1}^s is the value function of a seller without money at the beginning of the subsequent DM. Indeed, sellers are specialized in production, and hence do not need money since they do not consume in the DM. Consequently, sellers have no incentive to enter the DM with real balances, or to accumulate them, since they are costly to hold.

4 The decentralized market

We now describe the determination of the terms of trade in a bilateral match between a buyer holding the portfolio $z = f + e$ in the DM, and a seller without money. Agents bargain over (q, d) where the buyer receives $q \geqslant 0$ units of the search good produced by the seller in exchange for $d \in [0, z]$ units of money. The buyer may transfer to the seller d_f units of fiat money and/or d_e

[5] Although we do not need to formalize it explicitly for our analysis, we assume there are some agents (e.g. robbers) in the economy whose only goal, and activity, is to steal cash in the DM to consume goods in the CM. Any change in the measure of such agents, taken as given here, will affect the probability α of buyers and sellers to be stolen. However, the money stolen corresponds to a transfer of ressources from some agents to others, meaning the total stock of money brought and spent into the CM remains equal to the supply M.

units of electronic money. In order to give sellers an incentive to invest in the e-money technology, we adopt the proportional bargaining solution so they can extract a positive fraction of the total surplus. According to Kalai's bargaining solution, the buyer receives the fraction θ of the total surplus and the seller receives the remaining fraction $(1-\theta)$, where θ corresponds to the buyer's bargaining power.

The quantity of goods exchanged depends on the total amount of monetary units in the buyer portfolio, that may be composed of fiat, electronic, or both monies. By assumption, fiat money may be stolen whereas electronic money cannot. Indeed, if electronic purses are lost or stolen after a trade, buyers immediately report it to the issuer. In this case, the buyer is redeemed for the total amount stolen as the funds recorded on the electronic device will no longer be activated.

The terms of trade result from the maximization of the total surplus of each agent. When an agreement is reached, the buyers and sellers utilities are defined as follows:

$$\begin{aligned} u^b_{f,e} &= u\left[q\left(f,e\right)\right] + \left(1-\alpha\right) W^b\left(f-d_f, e-d_e\right) + \alpha W^b\left(0, e-d_e\right) \\ u^s_{f,e} &= -c\left[q\left(f,e\right)\right] + \left(1-\alpha\right) W^s\left(d_f, d_e\right) + \alpha W^s\left(0, d_e\right) \end{aligned}$$

The buyers utility $(u^b_{f,e})$ is the sum of three terms. The first term corresponds to the utility of consumption, the second and third terms to the continuation value in the CM when fiat money has not been stolen, and when it has (with probability α). Similarly, the sellers utility $(u^s_{f,e})$ is measured as the difference between the production costs and the continuation value of entering the CM with units of fiat and e-money. Without agreement, the buyers and sellers utilities satisfy:

$$\begin{aligned} u^b_0 &= \left(1-\alpha\right) W^b\left(f\right) + \alpha W^b\left(0\right) + W^b\left(e\right) \\ u^s_0 &= W^s\left(0\right) \end{aligned}$$

In the event fiat money is not stolen at the end of the DM, the buyer enters the CM with his units of fiat and e-money. Otherwise, he enters the CM with his electronic monetary units only. Without any transaction, sellers do not possess money.

Now, we can determine the trade surplus of each agent as the difference between the equations above:

$$\begin{aligned} S^b_{f,e} &= u\left[q\left(f,e\right)\right] - \left(1-\alpha\right) d_f - d_e \\ S^s_{f,e} &= -c\left[q\left(f,e\right)\right] + \left(1-\alpha\right) d_f + d_e \end{aligned}$$

where $S^b_{f,e}$ and $S^s_{f,e}$ are the buyers and sellers expected trade surplus respectively. Even if the buyer transfers the entire amount d_f of fiat money to the seller, only the fraction $(1-\alpha)$ of this amount is likely to be brought by the seller into the CM since it can be stolen with probability α. Indeed, when the buyer transfers money to the seller, he also transfers the risk of being stolen. As a result, the buyer transfers to the seller a certain amount of fiat money plus the risk attached to this currency, that reduces his own risk.

The determination of the terms of trade, when buyers can enter the market with fiat and electronic money, results from the following maximization program:

$$\begin{aligned} (q, d_f, d_e) &= \arg\max_{q,d_f,d_e} \left[u\left[q\left(f,e\right)\right] - \left(1-\alpha\right)d_f - d_e\right] \\ s.t.\ \left[u\left[q\left(f,e\right)\right] - \left(1-\alpha\right)d_f - d_e\right] &= \frac{\theta}{1-\theta}\left[-c\left[q\left(f,e\right)\right] + \left(1-\alpha\right)d_f + d_e\right] \\ d_f &\leqslant f,\ d_e \leqslant e \end{aligned} \tag{2}$$

According to this program, the buyer maximizes his utility of consuming the DM goods, net of the expected monetary units he transfers to the seller, and subject to the constraint that the buyers trading surplus is equal to $\theta/\left(1-\theta\right)$ times the sellers trading surplus, where the term $\theta/\left(1-\theta\right)$ corresponds to the relative bargaining power of agents. Additionally, the buyer cannot transfer to the seller more fiat or e-money than he has. From this program, we obtain the terms of trade:

$$\left(1-\alpha\right)d_f + d_e = z\left[q\left(f,e\right)\right] \tag{3}$$

with

$$z\left[q\left(f,e\right)\right] \equiv \left(1-\theta\right)u\left[q\left(f,e\right)\right] + \theta c\left[q\left(f,e\right)\right]$$

Equation (3) indicates that when the risk of being stolen (α) increases, the buyer must give a higher amount of fiat money (d_f) for the same quantity of goods than without the risk of theft. The additional amount of fiat money that the buyer gives to the seller corresponds to a *risk premium* that compensates the seller for the transfer of risk before entering the CM. Notice that this risk premium is only associated with the transfer of fiat money, but not with the transfer of e-money as it is secured. Consequently, in an unsafe economic environment, buyers obtain more goods in exchange for e-money than in exchange for fiat money. Substituting the terms of trade (3) into the buyers objective function (2), the buyers problem becomes:

$$q = \arg\max_q \theta\left[u\left[q\left(f,e\right)\right] - c\left[q\left(f,e\right)\right]\right]$$

$$s.t.\ (1-\alpha)\, d_f + d_e = z\left[q\left(f,e\right)\right]$$
$$d_f \leqslant f,\ d_e \leqslant e \tag{4}$$

The buyer's quantities consumed depend on his portfolio composition, and are determined so that he obtains the share θ of the total surplus. If the buyer carries enough monetary balances, (4) is not binding, and he consumes the optimal quantity q^*. However, if (4) is binding, he purchases the quantity $q(f,e) < q^*$. Thus, the terms of trade satisfy:

$$z\left[q\left(f,e\right)\right] = \min\left[z\left(q^*\right), (1-\alpha)\, f + e\right]$$

5 Buyers and sellers decisions

In this section, we study buyers portfolio choices, and sellers decisions to invest or not in the electronic payment terminal.

5.1 The buyers choice of money

Before entering the DM, according to equation (1), the buyer chooses the amount, and the type of money he wants to hold in his portfolio, from his DM value function that satisfies (Appendix 1):

$$\begin{aligned} V_t^b\left(f,e\right) &= \sigma\left(1-\Lambda\right)\theta\left[u\left(q_f\right) - c\left(q_f\right)\right] + \sigma\Lambda\theta\left[u\left(q_{f,e}\right) - c\left[q_{f,e}\right]\right] \\ &\quad + (1-\alpha)\, f + e + W_t^b\left(0,0\right) \end{aligned} \tag{5}$$

where $q_f \equiv q(f)$ is the quantity of search goods exchanged for fiat money, and $q_{f,e} \equiv q(f,e)$ is the quantity exchanged for both cash and e-money. Indeed, a buyer may be matched with a seller who does not accept e-money (with probability $1-\Lambda$), or a seller who accepts the two monies (with probability Λ). The last two terms result from the linearity of the value function, and correspond to the continuation value in the CM when no transaction occurs. The buyer does not expect to enter the CM with his initial portfolio, but only with the expected units of fiat money that will not be stolen, yet he will bring all of his units of electronic money in the CM.

Substituting (5) into (1), the buyers real balance choice is given by (Appendix 2):

$$\Psi\left(f,e\right) = \max_{f,e\geqslant 0}\left\{\begin{array}{c} -i_\alpha f - ie \\ +\sigma\theta\left\{(1-\Lambda)\left[u\left(q_f\right) - c\left(q_f\right)\right] + \Lambda\left[u\left(q_{f,e}\right) - c\left[q_{f,e}\right]\right]\right\} \end{array}\right\} \tag{6}$$

where $i = \frac{\gamma-\beta}{\beta}$, and $i_\alpha = \frac{\gamma-(1-\alpha)\beta}{\beta} = i + \alpha$ represent the holding cost of electronic and fiat money, respectively.

The first two terms of (6) correspond to the costs incurred by the buyer when he carries f real units of fiat money and e real units of electronic money in the subsequent DM. The costs of holding fiat or e-money are different. The third term of (6) represents the buyers share of the expected trade surplus if he enters the market with fiat money only, or if he enters with both monies. Both surpluses depend on the fraction Λ of sellers who invested in the e-payment technology.

Lemma 1: When the risk of theft is strictly positive, the holding cost of fiat money is higher than that of e-money.

Indeed, the holding cost of fiat money (i_α) is the sum of two costs: the opportunity cost (i) of both fiat and e-money, plus the insecurity cost of trading with cash (α).

When $i > 0$, both monies are costly to hold, so buyers will never accumulate more real balances than they need in the DM ($z \leqslant z(q^*)$). According to (6), the buyer chooses his money balances ($f \geqslant 0$, $e \geqslant 0$) to maximize his share of the expected trade surplus in the DM, net of the holding costs of each type of money. The buyers objective function is strictly concave, so a solution exists. The first order (necessary and sufficient) conditions associated with (6) are (Appendix 3):

$$\frac{-i_\alpha}{\sigma\theta} + (1-\Lambda)\left[\frac{(1-\alpha)\left(u'\left(q_f\right) - c'\left(q_f\right)\right)}{(1-\theta)\,u'\left(q_f\right) + \theta c'\left(q_f\right)}\right] + \Lambda\left[\frac{(1-\alpha)\left(u'\left(q_{f,e}\right) - c'\left(q_{f,e}\right)\right)}{(1-\theta)\,u'\left(q_{f,e}\right) + \theta c'\left(q_{f,e}\right)}\right] \leqslant 0 \qquad (7)$$

$$\frac{-i}{\sigma\theta} + \Lambda\left[\frac{u'\left(q_{f,e}\right) - c'\left(q_{f,e}\right)}{(1-\theta)\,u'\left(q_{f,e}\right) + \theta c'\left(q_{f,e}\right)}\right] \leqslant 0 \qquad (8)$$

According to equations (7) and (8), a buyer brings positive real balances into the DM up to the point where he equalizes the holding cost of a marginal unit of each type of money with its marginal return, i.e. with the expected increase in the trade surplus. The marginal return of each currency depends both on the risk of theft of fiat money (α), and the degree of acceptability of electronic money by sellers (Λ).

5.2 The sellers investment decision

At the beginning of the DM, sellers choose whether to invest in the technology to accept electronic money in parallel to cash. This choice results from the following maximization program:

$$\max\left\{\sigma(1-\theta)\left(u\left(q_f\right) - c\left(q_f\right)\right), -I + \sigma(1-\theta)\left(u\left(q_{f,e}\right) - c\left(q_{f,e}\right)\right)\right\} \qquad (9)$$

According to (9), if a seller does not invest in the reading terminal, he can only sell his goods in exchange for fiat money, and the quantity exchanged is q_f. In this case, he receives the fraction

$(1-\theta)$ of the total surplus obtained with fiat money. On the contrary, a seller who chooses to invest (at cost I) in the new technology can exchange a quantity $q_{f,e}$ of goods for fiat and e-money, and receive the fraction $(1-\theta)$ of the total surplus obtained with both monies. From (9), the fraction Λ of sellers who decide to invest in the e-payment terminal satisfies:

$$\Lambda \begin{cases} =1 \\ \in (0,1) \\ =0 \end{cases} \text{if } -I+\sigma(1-\theta)\left(u\left(q_{f,e}\right)-c\left(q_{f,e}\right)\right) \begin{cases} > \\ = \\ < \end{cases} \sigma(1-\theta)\left(u\left(q_f\right)-c\left(q_f\right)\right) \tag{10}$$

Now, we are able to study the different equilibria of the model that arise from buyers and sellers interactions.

6 Monetary equilibria

In this section, we describe the different Nash equilibria arising from the previous individual choices. There are strategic complementarities as the decision of each agent (buyer/seller) depends on the decision of the other. Buyers choose the composition of their portfolio based on their expectations of the fraction $\Lambda \in [0,1]$ of sellers who will accept e-money, and sellers decide to invest in the payment technology based on their expectations about the type of money buyers will use.

We consider three successive economic environments: one where all sellers accept electronic money $(\Lambda = 1)$, one where all sellers refuse e-money $(\Lambda = 0)$, and one where a fraction of sellers accept e-money $(\Lambda \in (0,1))$. In each of these environments, we determine the optimal portfolio choice of the buyer, and then according to this choice, we check if it is optimal for the seller to invest in the new payment technology. This second step makes Λ an endogenous variable of our model.

Definition: A steady-state equilibrium is a list (q, f, e, Λ) *that satisfies* (3), (7), (8) *and* (10).

6.1 All sellers accept electronic money

First, we study the equilibrium such that all sellers have invested in the e-money technology $(\Lambda = 1)$. Consequently, two means of payment can be accepted to settle transactions: fiat money and e-money. From (7) and (8) with $\Lambda = 1$, the equilibrium output, if traded, is the solution to:

$$\frac{i_\alpha}{\sigma\theta} = \frac{(1-\alpha)\left(u'\left(q^1_{f,e}\right) - c'\left(q^1_{f,e}\right)\right)}{(1-\theta)\,u'\left(q^1_{f,e}\right) + \theta c'\left(q^1_{f,e}\right)}$$

$$\frac{i}{\sigma\theta} = \frac{u'\left(q^1_{f,e}\right) - c'\left(q^1_{f,e}\right)}{(1-\theta)\,u'\left(q^1_{f,e}\right) + \theta c'\left(q^1_{f,e}\right)}$$

where the exponent "1" refers to an equilibrium where all sellers accept e-money.

These two equations establish the conditions for fiat money and electronic money to be valued. Buyers are indifferent to both monies when these equations are equal, i.e if and only if $\alpha = 0$; otherwise e-money is prefered to cash. Indeed, when there is no risk of theft of fiat money, and when both currencies are accepted by all sellers, buyers are indifferent to holding cash or using electronic money. An additional unit of one or the other type of money involves the same marginal increase in the buyers surplus. However, when the risk of theft (α) is strictly positive, it is not rational for a buyer to use fiat money given that electronic money, which is safer and less expensive to use, is always accepted. Since e-money has a higher marginal return, and a lower opportunity cost than fiat money, the equilibrium is such that only electronic money is used. In this equilibrium, fiat money is no longer used and valued, and the quantity exchanged with electronic money is higher than the quantity that would have been exchanged with cash. However, when $i > 0$, the quantity traded is less than the optimal quantity (q^*).

The monetary authority can drive the holding cost of e-money to zero. Indeed, when $\gamma = \beta$, the opportunity cost of holding money (i) is zero, whereas the cost of insecurity of fiat money remains positive for all $\alpha > 0$. Therefore, if the opportunity cost of e-money is zero ($i = 0$), at the Friedman rule buyers will hold enough electronic money to buy the optimal quantity of output q^*.

When $\Lambda = 1$, and $\alpha > 0$, the buyers choice of money holdings, and the quantities traded, are such that:

$$\Lambda = 1 \Longrightarrow \begin{cases} f^1 = 0 \\ z^1 = e^1 = z\left(q^1_e\right) \\ q = q^1_e < q^* \end{cases}$$

We now analyze the sellers investment decisions when buyers use electronic money. According to (10), when $\Lambda = 1$, it is optimal for a seller to invest in the e-payment technology if:

$$I < G^1_{f,e} \equiv \sigma(1-\theta)\left[S^1_{f,e} - S^1_f\right] \tag{11}$$

where $S^1_{f,e} = u\left(q^1_{f,e}\right) - c\left(q^1_{f,e}\right)$, $S^1_f = u\left(q^1_f\right) - c\left(q^1_f\right)$, and $G^1_{f,e}$ is the gain obtained by a seller if he accepts both fiat and e-money instead of fiat money only, in an environment where all other sellers accept both monies.

Equation (11) indicates that when the expected increase in the total surplus resulting from the use of electronic money is higher than the fixed investment cost ($G^1_{f,e} > I$), the seller will rationally choose to invest in the new payment technology. In (11), q^1_f is the output produced by a seller who did not invest in e-money equipment while all the other merchants have invested. Sellers who invested will exchange the quantity $q^1_{f,e}$ as they can accept both monies. The quantity q^1_f satisfies $(1-\alpha)\, f^1 = z\left(q^1_f\right)$, and since from (7–8) $f^1 = 0$ when $\Lambda = 1$, the quantity traded $q^1_f = 0$. Consequently, since only e-money is valued and used, $q^1_{f,e} = q^1_e$, and $G^1_{f,e} = S^1_e > 0$. Therefore, it is optimal for a seller to invest if the cost is strictly smaller than the gain resulting from the use of e-money. The sellers best response function is:

$$\left.\begin{array}{c} q = q^1_e \\ 0 \leqslant I < G^1_{f,e} \end{array}\right\} \Longrightarrow \Lambda = 1$$

From the buyers and sellers choices, there exists an equilibrium (pure e-money equilibrium) such that electronic money becomes the universally accepted means of payment whereas fiat money disappears from the economy. This equilibrium corresponds to point E (for “Electronic”) in Figure 2.

Proposition 1: When all sellers accept both fiat and electronic money, the only equilibrium is such that cash does not circulate in the economy when $\alpha > 0$; e-money becomes the only payment instrument.

6.2 No seller accepts electronic money

We study the equilibrium such that no seller has invested in the reading terminal ($\Lambda = 0$). Consequently, sellers can only trade if buyers use fiat money. From (7), with $\Lambda = 0$, electronic money has no value, and the equilibrium output exchanged for cash is the solution to:

$$\frac{i_\alpha}{\sigma\theta} = \frac{(1-\alpha)\left(u'\left(q^0_f\right) - c'\left(q^0_f\right)\right)}{(1-\theta)\, u'\left(q^0_f\right) + \theta c'\left(q^0_f\right)}$$

where the exponent "0" refers to an equilibrium where no seller accepts e-money. This equation can be rewritten:

$$\frac{u'\left(q^0_f\right)}{c'\left(q^0_f\right)} = \frac{\theta\left[i_\alpha + \sigma(1-\alpha)\right]}{\theta\left[i_\alpha + \sigma(1-\alpha)\right] - i_\alpha}$$

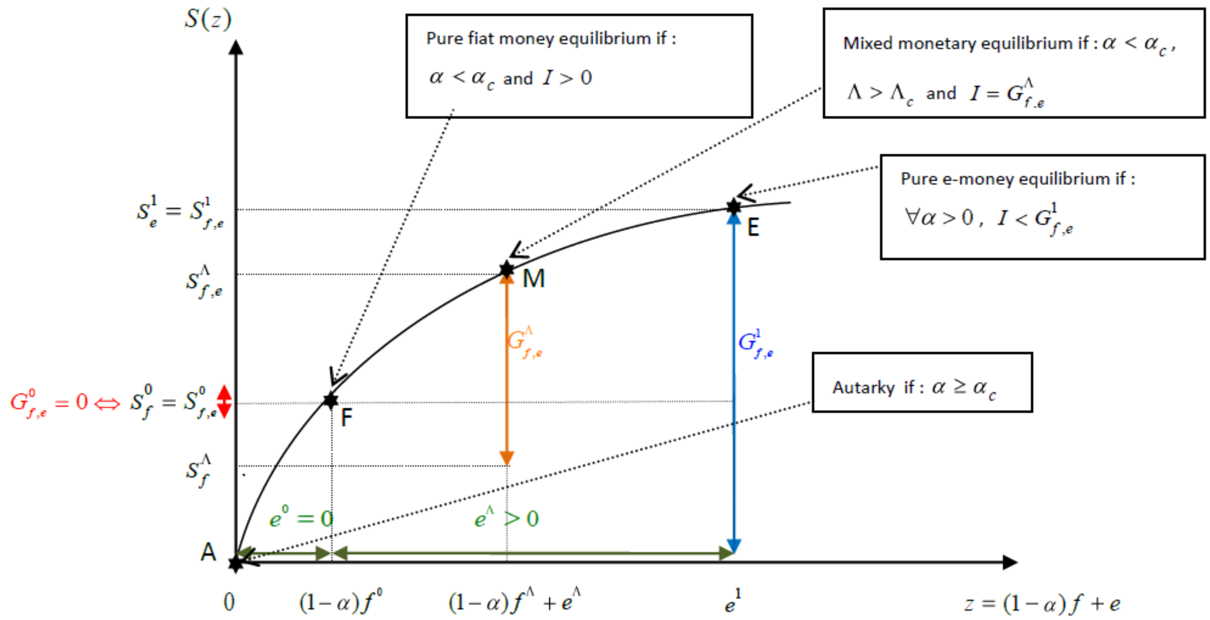

Figure 2: Monetary Equilibria

Lemma 2: When $\Lambda = 0$, there is a critical value α_c for the risk of theft such that if $\alpha \geqslant \alpha_c$, then $q_f^0 = 0$ and $f^0 = 0$, i.e. fiat money is no longer valued.

The critical value α_c above which buyers no longer use cash satisfies:

$$\alpha_c = \frac{\sigma\theta - i(1-\theta)}{\sigma\theta}$$

so for fiat money to be valued, the risk of theft must be sufficiently small ($\alpha < \alpha_c$).

Buyers portfolio choice depends on the level of risk of using fiat money, but also on buyers expectations about the number of sellers accepting electronic money. When no seller accepts it, e-money cannot be used as a medium of exchange. Indeed, according to equation (8), if $i > 0$, $0 \leqslant \alpha < \alpha_c$, and $\Lambda = 0$, no buyer will use e-money, even if the e-purse is obtained for free.[6]

In an environment where no seller accepts e-money, and where the risk of theft is small enough, the only equilibrium is a pure fiat money equilibrium. Indeed,

$$\Lambda = 0, \alpha < \alpha_c \implies \begin{cases} e^0 = 0 \\ z^0 = f^0 > 0 \\ (1-\alpha) f^0 = z\left(q_f^0\right) \\ q = q_f^0 < q^* \end{cases}$$

[6] When $i = 0$, and $0 < \alpha < \alpha_c$, an equilibrium with $\Lambda = 0$ is impossible. The only equilibrium is a pure e-money equilibrium (see section 6.1).

Furthermore, (9) implies that it is optimal for sellers not to invest in the e-money technology if:

$$I > G_{f,e}^{0} \equiv \sigma (1-\theta) \left[S_{f,e}^{0} - S_{f}^{0}\right] \tag{12}$$

where $S_{f,e}^{0} = u\left(q_{f,e}^{0}\right) - c\left(q_{f,e}^{0}\right)$, $S_{f}^{0} = u\left(q_{f}^{0}\right) - c\left(q_{f}^{0}\right)$, and $G_{f,e}^{0}$ is the gain for a seller accepting both monies when all other sellers accept cash only.

Equation (12) indicates that when the expected increase in the total surplus resulting from the acceptance of e-money is lower than the fixed investment cost ($G_{f,e}^{0} < I$), the seller will rationally choose not to invest in the new payment technology. In (12), $q_{f,e}^{0}$ is the output produced by a seller who has invested in the e-money technology when all the other merchants have not. Sellers who only accept fiat money will exchange the quantity q_{f}^{0}. From (7–8), since $e^{0} = 0$ when $\Lambda = 0$, the quantity traded $q_{f,e}^{0} = q_{f}^{0}$. Consequently, $S_{f,e}^{0} = S_{f}^{0}$ and $G_{f,e}^{0} = 0$. Therefore, it is optimal for a seller not to invest if the cost is strictly positive. The sellers best response function is:

$$\left.\begin{array}{c} q_{f,e}^{0} = q_{f}^{0} \\ I > 0 \end{array}\right\} \Longrightarrow \Lambda = 0$$

For the same fundamentals as the previous case, there also exists an equilibrium such that e-money is not used, and fiat money keeps its role of universally accepted means of payment (pure fiat money equilibrium). The quantities traded in a pure fiat money equilibrium are lower than in the pure electronic money equilibrium given the risk of theft associated with cash. This equilibrium corresponds to point F (for "Fiat") in Figure 2.

Proposition 2: When e-money is less costly to hold than fiat money, there exist an equilibrium such that e-money does not circulate in the economy; fiat money remains the only payment instrument. [The quantity of goods exchanged in a pure fiat money equilibrium is lower than in a pure e-money equilibrium].

6.3 Some sellers accept e-money

We have studied the existence of pure monetary equilibria such that only one type of money is used as a medium of exchange. However, is it possible to have the coexistence of fiat and e-money in the economy, such that a fraction $\Lambda \in (0,1)$ of sellers accept both fiat and e-money, whereas a fraction $1-\Lambda$ of sellers only accept fiat money?

Again, buyers expectations about the sellers level of investment are crucial to encourage them to use e-money in parallel with cash. From (8), the equilibrium output $q_{f,e}^{\Lambda}$ exchanged for both cash and e-money satisfies:

$$\frac{i}{\sigma\theta} = \Lambda \left[\frac{u'\left(q_{f,e}^{\Lambda}\right) - c'\left(q_{f,e}^{\Lambda}\right)}{(1-\theta)\,u'\left(q_{f,e}^{\Lambda}\right) + \theta c'\left(q_{f,e}^{\Lambda}\right)} \right] \tag{13}$$

where the exponent "Λ" refers to an equilibrium where only a fraction $\Lambda \in (0,1)$ of sellers accept electronic money. The quantity traded when only cash is accepted (q_f^{Λ}) is the solution of:

$$\frac{i_\alpha}{\sigma\theta} = (1-\Lambda)\left[\frac{(1-\alpha)\left(u'\left(q_f^{\Lambda}\right) - c'\left(q_f^{\Lambda}\right)\right)}{(1-\theta)\,u'\left(q_f^{\Lambda}\right) + \theta c'\left(q_f^{\Lambda}\right)}\right] + \Lambda\left[\frac{(1-\alpha)\left(u'\left(q_{f,e}^{\Lambda}\right) - c'\left(q_{f,e}^{\Lambda}\right)\right)}{(1-\theta)\,u'\left(q_{f,e}^{\Lambda}\right) + \theta c'\left(q_{f,e}^{\Lambda}\right)}\right] \tag{14}$$

After rearrangements, equation (13) involves:

$$\frac{u'\left(q_{f,e}^{\Lambda}\right)}{c'\left(q_{f,e}^{\Lambda}\right)} = \frac{\theta\left[i + \sigma\Lambda\right]}{\theta\left[i + \sigma\Lambda\right] - i}$$

Therefore, we determine the critical value Λ_c above which buyers decide to enter the DM with e-money:

$$\Lambda_c = \frac{i}{\sigma}\frac{(1-\theta)}{\theta}$$

Below this threshold, there would be an insufficient number of sellers accepting e-money to encourage buyers to use it. This critical value is an increasing function of the interest rate i.

We show (Appendix 4) that for any $\Lambda \in (\Lambda_c, 1)$, there always exist a quantity $q_{f,e}^{\Lambda} > 0$ and $q_f^{\Lambda} > 0$ such that (13) and (14) are simultaneously satisfied. Consequently, when $\Lambda \in (\Lambda_c, 1)$, some buyers may enter the market with e-money, as well as cash. The buyers portfolio composition in a mixed monetary equilibrium is defined by:

$$\Lambda \in (\Lambda_c, 1) \Longrightarrow \begin{cases} e^{\Lambda} > 0,\ f^{\Lambda} > 0 \\ (1-\alpha)\, f^{\Lambda} + e^{\Lambda} = z\left(q_{f,e}^{\Lambda}\right) \\ (1-\alpha)\, f^{\Lambda} = z\left(q_f^{\Lambda}\right) \\ q_f^{\Lambda} < q_{f,e}^{\Lambda} < q_e^1 < q^* \end{cases}$$

Moreover, according to (9), a seller is indifferent between trading the quantity $q_{f,e}^{\Lambda}$ or q_f^{Λ} if:

$$I = G_{f,e}^{\Lambda} \equiv \sigma(1-\theta)\left[S_{f,e}^{\Lambda} - S_f^{\Lambda}\right]$$

where $S_{f,e}^{\Lambda} = u\left(q_{f,e}^{\Lambda}\right) - c\left(q_{f,e}^{\Lambda}\right)$, $S_f^{\Lambda} = u\left(q_f^{\Lambda}\right) - c\left(q_f^{\Lambda}\right)$, and $G_{f,e}^{\Lambda}$ is the gain for a seller if he accepts both monies instead of cash only, when a fraction of sellers accept e-money.

Proposition 3: When $G_{f,e}^0 < I = G_{f,e}^{\Lambda} < G_{f,e}^1$, and $\Lambda \in (\Lambda_c, 1)$, *there exists an equilibrium such that some sellers accept only fiat money whereas others accept both fiat and e-money, and buyers use both.*

The fact that a mixed monetary equilibrium exists is quite an intuitive result. Indeed, payment with electronic money allows exchanging a larger amount of goods than with cash. Therefore, if buyers anticipate that e-money may be accepted by some sellers, all of them will decide to possess e-money in addition to cash. This equilibrium corresponds to point M (for "Mixed") in Figure 2.

In summary, different multiplicities may appear, depending on the value of the investment cost. In Figure 2, the surplus in a mixed monetary equilibrium is lower than the surplus in the pure e-money equilibrium. When the value of I satisfies the sellers investment condition for a mixed equilibrium to exist (point M), then the sellers investment condition for a pure electronic money equilibrium to exist (point E) is also satisfied. Of course, the equilibrium where only fiat money is accepted always exist as long as the risk of theft of fiat money is low enough. Therefore, three monetary equilibria may coexist, where no seller, all sellers, or a fraction of them choose to invest in the new technology. However, if the investment cost is too high for the pure electronic money equilibrium to exist, then the economy will end up in a pure fiat money equilibrium (point F). In addition, if the investment cost is zero, then electronic money dominates fiat money, and the only equilibrium is with electronic money. Indeed, e-money is entirely secured, allowing for an exchange of a larger amount of goods.

7 E-purse adoption in some developed countries

The first electronic purse initiatives, which appeared in Europe in the early 1990s, were followed by the launch of the Asian e-purse in the late 1990s, and by the prepaid cards in the United States in the 2000s. These electronic money projects have not always been successful since the failure of the European e-purse differ from the success of the Asian and American cash cards. To understand this, it is worth considering the initial objectives of e-money issuers and their development strategies to assess the role of each variable (α, I and Λ) of our model.

The aim of European e-purse was ambitious. Developers wanted electronic money to progressively replace cash in the retail sector. Thus, European e-purses are "digitalized" physical purses which offer no additional service compared to cash. In contrast, Asian prepaid cards, launched in 1997 in Hong Kong, and in 1998 in Japan and Singapore, were not designed to replace cash in transactions with merchants, but to replace cash in vending machines (i.e. where the need to have the right change is more binding). Specifically, Asian e-purse appeared in the form of payment cards necessary to buy mass transit tickets, and their distribution was based on preferential pricing.

Therefore, users have simply replaced the use of cash in payment machines at the stations entrance. It was only after the development of a critical mass of users that the use of Asian e-purse has been extended, first to the markets in connection with the rail transport (such as parking), then to the merchants near the train stations, and finally to other markets. Consequently, the Asian e-purses have been developed as transport cards, which distinguished them from the European physical purses. In addition, they integrate contactless technology. Regarding the US prepaid cards, they also provide a new service compared to physical wallets. In fact, currently, one American in five does not have any bank account (i.e. is "underbanked"). Thus, prepaid cards issued by the bank card networks enable this particular population to have access to a form of electronic payment. Again, the purpose of the e-purse is not to replace the use of cash, which is deeply rooted in the US, but to provide a new service to some consumers (i.e. new access to an electronic payment method).

Developers of electronic money face three problems which correspond to the three main variables of our model: (*i*) its advantage in terms of safety, compared to cash (α), (*ii*) the investment cost incurred by sellers (I), or equivalently the net gain resulting from this investment (G), and (*iii*) its degree of acceptance by merchants (Λ).

The primary advantage of an e-purse is the possibility to block the card in the event of loss or theft. In Asia, and in the United States, the owners of e-purses have a guarantee against the risk of loss or theft if their e-purse is personalized. This guarantee of refund cancels the risk of theft associated with the holding of electronic balances. Unlike the general case described in our model, European e-purses do not offer this type of guarantee to buyers. Therefore, e-money offers nothing more than cash (the reloadable smart cards are exposed to the same risks as cash), except the time saved in transactions, offset by a loss of time equivalent due to reloading. However, sellers who receive e-money payments run no risk of loss or theft when they use e-money, compared to cash.

Sellers investment in the new payment technology was not always profitable. The distribution strategy of European e-purses was to provide the buyers with the smart card for free, or at negligable cost, while the sellers had to pay a lump sum to acquire the payment terminal, plus commissions on transactions. Because sellers did not benefit from any captive market, no one invested in the payment terminal since they could continue to accept cash. Instead, in Asia, consumers had to purchase prepaid cards at a fixed cost. However, their advantages compared to fiat money (safety, no need for change, discounts ...) were valued by consumers. Additionally, providers subsidized the transportation companies. Thus, the issuer of the payment system fully incurred the investment cost, and then charged commissions based on transactions in return for the service provided. Finally, US e-purses are fully compatible with credit card terminals, since they are issued by the same providers, so merchants have no additional investment costs.[7]

Finally, the main concern of a buyer in possession of a e-purse is the risk of not being able to use e-money if some merchants do not accept it. In Europe, the choice of switching to electronic money was left to the discretion of agents who had received information campaigns based on the convenience, and speed of payments with e-money. However, to receive e-money, European merchants had to acquire a new payment terminal. As buyers were reluctant to use prepaid cards, not knowing if sellers would accept them, most sellers did not acquire the electronic payment terminal. This lack of coordination induced an adoption level close to zero ($\Lambda \approx 0$), while Asian projects have been developed on the captive markets of mass transit. This strategy has made the e-purse usage essential ($\Lambda \approx 1$), and not optional like in Europe.[8] In the United States, prepaid cards issued by the bank card networks guarantee an even wider use since they are compatible with the same payment terminals as those used for traditional bank cards. Thus, the holder of e-money knows that he can use it at all merchants who accept credit cards. Consequently, the probability of acceptance of US prepaid cards is very high, whatever the type of market ($\Lambda \approx 1$).

The following table compares the electronic purse systems developed in Europe, Asia, and the United States, with the parameter values of our model, and confirms the predictions of our model.

Variables	Model	Europe	Asia	United-States
Probability of loss of e-money balances	zero	Positive (*)	zero	zero
I Sellers' investment cost	$I \geq 0$	$I > 0$ high	$I \simeq 0$ low	$I = 0$ zero
G Sellers' gain of electronic money acceptance	$G \geq 0$	$G = 0$ Very low	$G > 0$ high	$G > 0$ high
Λ (endogenous variable) Probability that merchants accept electronic money	$\Lambda \in [0,1]$	$\Lambda \approx 0$ A few markets	$\Lambda = 1$ In captive market	$\Lambda = 1$ In all markets

(*) only for buyers. E-money transfers are fully secured for merchants

[7] Like credit cards, merchants pay a fee per transaction, and consumers also pay commissions for the service associated with their prepaid cards.

[8] In Hong Kong, the Octopus card has mainly benefited from the monopoly of the Octopus Company, sole supplier of the new payment method, which equipped in a few months all the turnstiles.

8 Conclusion

In order to better understand why the launching of a new type of payment instrument may succeed or fail, it is important to analyze the crucial role of buyers, and sellers decisions. It is also necessary to emphasize their interactions as the markets of payment instruments are two-sided markets. In our model, we examine the policy of replacing fiat money with e-money by considering the direct and indirect costs and benefits of each type of money. More precisely, we determine monetary equilibria as a function of three main variables: the safety level of the monetary instrument that can be used as a medium of exchange, the cost of investment in a new e-payment terminal, and the sellers e-money adoption level.

Three types of equilibria can emerge. When all sellers can accept the two types of money, the unique equilibrium is such that only electronic money is used, meaning fiat money is no longer valued. However, when no seller accepts e-money, the latter is never valued, and fiat money may or may not be valued, depending on the risk of theft. We also demonstrate the existence of an equilibrium where both fiat and electronic money are used as a means of payment. In that case, all buyers hold both fiat and electronic monetary units, although only a portion of sellers accept e-money. This equilibrium coexists with a pure fiat money and pure e-money equilibrium. These results come from the strategic complementarities between buyers and sellers in the adoption process of a new currency.

The conclusions of our model can be demonstrated by past experiences. In Japan or Singapore, where electronic purses are widely used, we can see the importance of prescribed uses in the adoption process of e-money. In these countries, not only the purchase of mass transit tickets must be made with electronic money, but it is the payment instrument issuer that fully incurs the investment cost. In the US, the recent success of prepaid cards usage is linked to the fact that it is compatible with the payment terminals used by the bank industry for debit and credit card transactions. In this situation, merchants have no additional investment cost to incur. Finally, in Europe, the failure of the electronic purse is probably due to the lack of existence of a captive market, high investment costs, and no “critical mass” of users necessary for a new payment instrument to be widely adopted.

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